Stellar masses: the comparison of theoretical predictions and measurement data

B.V.Vasiliev

Abstract

The Euler equation has been accepted as the basic postulate of stellar physics long before the plasma physics was developed. The existence of electrical interaction between particles of interstellar plasma poses the question, how this interaction must be accounted for. We argue that the right way is in formulation of a new postulate. On the base of the new postulate, the theory of a hot star interior is developed. Using this theory we obtain the distribution of stars over their masses and mass-radius, mass-temperature and mass-luminosity dependencies. All these theoretical predictions are in a good agreement with the known measurement data, which confirms the validity of the new postulate.

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1 Introduction

The astrophysics uses the Euler equation

$$\gamma\mathbf{g} - \nabla P = 0 \tag{1}$$

as its basic postulate at a stellar interior balance description. It contends that a gravity force acting on substance inside star are balanced by induced pressure gradient. During a long time this claim seemed so obvious that nobody doubted its applicability to the description of stellar substance equilibrium. Later on, as the plasma physics has been developed, one could see that the particles of stellar substance were possessing not only masses but also electrical charges. The gravity can induce a redistribution of electrical charges in plasma inside a star, i.e. produce an electric polarization of plasma. The existence of this polarization must be taken into account by formulating the equilibrium condition of stellar interior. The force acting in a polarized substance is proportional to its polarization $\mathfrak{P}$ and gradient $\mathfrak{P}$ [2]. So the equilibrium equation taking into account this force has the form:

$$\gamma\mathbf{g} + 4\pi\mathfrak{P}\nabla\mathfrak{P} - \nabla P = 0. \tag{2}$$

The Eq.(1) describes the balance of non-polarized matter. One could suppose that under certain conditions the equilibrium of plasma in a star can be reached in another way, when a gravity force is balanced by an induced electrical force. Therefore, instead of the Eq.(1) we postulate an alternative solution of this problem:

$$\gamma\mathbf{g} + 4\pi\mathfrak{P}\nabla\mathfrak{P} = 0. \tag{3}$$

The electrodynamic says that an electrical polarization can be described as an appropriate bonded charge distribution

$$div\mathfrak{P} = \rho_{bond}, \tag{4}$$

and the equilibrium equation in this case can be formulated as

$$\gamma\mathbf{g} + \rho_{bond}\ \mathbf{E} = 0 \tag{5}$$

where the intensity $\mathbf{E}$ is determined by equation:

$$4\pi\rho_{bond} = div\ \mathbf{E}. \tag{6}$$

It was calculated before [1] that the equilibrium equation Eq.(5) can be realized at condition of very high temperature and density of plasma only, i.e. in a central region of a star. As the gradient of pressure is absent at the equilibrium condition (Eq.(5)), it is not difficult to conclude that a core with a constant particle (electron) density in plasma [1]

$$\eta_\star = \frac{16}{9\pi}\frac{(Z+1)^3}{r_B^3}, \tag{7}$$

at constant temperature

$$\mathbb{T} = \left(\frac{10}{\pi^4}\right)^{1/3}(Z+1)\frac{\hbar c}{kr_B} \approx 2\cdot 10^7(Z+1)\ K. \tag{8}$$

must be placed in centrum of a star.

Its mass must be equal to [1]

$$\mathbb{M} = 1.5^6\left(\frac{10}{\pi^3}\right)^{1/2}\left(\frac{\hbar c}{Gm_p^2}\right)^{3/2}\left(\frac{Z}{A}\right)^2 m_p \approx 6.47\ M_{Ch}\left(\frac{Z}{A}\right)^2, \tag{9}$$

where $M_{Ch} = \left(\frac{\hbar c}{Gm_p^2}\right)^{3/2} m_p = 3.71\cdot 10^{33} g$ is the Chandrasekhar mass, A and Z are the averaged mass and charge numbers of atomic nuclei from which the plasma is composed, r_B is the Bohr radius, m_p is the proton mass. The radius of a star core

$$\mathbb{R} = \frac{(3/2)^3}{2}\left(\frac{10}{\pi}\right)^{1/6}\left(\frac{\hbar c}{Gm_p^2}\right)^{1/2}\frac{r_B}{(Z+1)A/Z}. \tag{10}$$

must be equal approximately to 1/10 of an external radius R_0 of a star. It can be seen, that all main parameters of a star core are depending on a chemical composition (A/Z and $Z+1$) of its plasma only.

A substance in region above a star core must be three order of magnitude more rarefied. This region can be named as an atmosphere of a star.

A testing of the validity of a fundamental postulate of a theory is a standard procedure which was developed by the scientific community since G.Galileo: the laws must by formulated by means of standard mathematical methods on the base of the tested postulate. Then one must check empirically that the Nature "obeys" these laws and as result to confirm

the validity of the fundamental postulate. In our case on a base of the postulate (Eq.(3)) (or in a more convenient form Eq.(5)), the theoretical conclusions about main characteristics of a star core have been deduced (Eq.(7),Eq.(8),Eq.(9), Eq.(10)). But the astronomers cannot measure the internal characteristic of stars directly. They can measure under some condition the integral and surface characteristics of stars - masses, radii, surface temperatures, luminosities. To have the possibility to compare theoretical predictions with measured characteristics of stars, we must calculate corresponding parameters of a star atmosphere. The solution is possible if one considers a stationary stable state of a star as its equilibrium one. A hot star generates an energy continuously and radiates it from its surface. This radiation is non-equilibrium relatively to a star, but it can be stationary stable. The substance of a star can be considered to have an equilibrium state in adiabatic conditions because the existing exchange of energy between the radiation subsystem and the subsystem of substance is permanent and doesn't induce an alteration of state of the substance. Hence the description of a state of a star substance can be based on a consideration of an equilibrium condition of a hot plasma or, in the first approximation, on a consideration of an equilibrium of an ideal gas in adiabatic conditions.

2 The equilibrium state of substance of a star interior and stellar masses

An ideal gas in a volume without gravitation exists in equilibrium state at a pressure equalization, i.e. at an equalization of temperature T and particle density n over volume. The constancy of chemical potential μ is mandatory requirement of this equilibrium state.

If a different parts of a system have different temperatures in an equilibrium state, the mandatory requirement of equilibrium adds up to [2]:

$$\frac{\mu}{kT} = const \tag{11}$$

As thermodynamical (statistical) part of chemical potential of an ideal gas [2]:

$$\mu_T = kT\ ln\left[\frac{n}{2}\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\right], \tag{12}$$

the density of ideal gas in equilibrium state must depend on temperature

$$n \sim T^{3/2}. \tag{13}$$

The chemical potential of a system at gravity action [2]

$$\mu = \mu_T + E_G \tag{14}$$

where E_G is the gravitational energy of particle. Hence the equilibrium state needs a fulfilment of additional condition:

$$\frac{GM_r m}{rkT_r} = const \tag{15}$$

(where m is the mass of a particle, M_r is the mass of star substance inside spherical volume with radius r, T_r is the temperature on this surface) or

$$M_r \sim rkT_r. \tag{16}$$

If we suppose that temperature inside star behaves as a power with exponent x, its value on the radius r

$$T_r = T_\star \left(\frac{\mathbb{R}}{r}\right)^x \tag{17}$$

and accordingly the particle density

$$n_r = \eta_\star \left(\frac{\mathbb{R}}{r}\right)^{3x/2}. \tag{18}$$

From Eq.(16), at equalizing of the exponent at r in the left and right parts, one obtains $x = 4$, i.e.

$$n_r = \eta_\star \left(\frac{\mathbb{R}}{r}\right)^6 \tag{19}$$

and

$$T_r = \mathbb{T} \left(\frac{\mathbb{R}}{r}\right)^4 \tag{20}$$

Hence the mass of star atmosphere

$$M_A = 4\pi \int_{\mathbb{R}}^{R_0} m'\eta_\star \left(\frac{\mathbb{R}}{r}\right)^6 r^2 dr \approx \frac{4\pi}{3} m'\eta_\star \mathbb{R}^3 = \mathbb{M} \tag{21}$$

is equal to mass of its core (accurate to $\frac{\mathbb{R}^3}{R_0^3} \approx 10^{-3}$), where m' is the mass of star plasma related to one electron.

This claim is in a good agreement with measurement data of Sun properties. The core mass of the Sun was calculated before [6] on a base of data of measurement of natural frequencies of seismical sunny oscillations [5]. According to this data the core mass of the Sun equals to $9.6\ 10^{32}\ g$ (it is 0.48 mass of the Sun), i.e. for the Sun $M_A \approx \mathbb{M}$ really.

As a result the full mass of a star

$$M = 2\mathbb{M} \approx 12.9\ M_{Ch} \left(\frac{Z}{A}\right)^2 \tag{22}$$

depends of the ratio A/Z only, and for comparison this estimation with measurement data we need the chemical composition of distant stars, which is unknown. But some predictions in this direction are possible. At first, there must be no stars with masses exceeding the Sun mass more than one and a half orders, because it corresponds to the limiting mass of stars consisting from hydrogen with $A/Z = 1$. Secondly, though the neutronization process makes neutron-excess nuclei stable, there is no reason to suppose that stars with A/Z which are essentially larger than few units (and with mass almost in hundred times less than hydrogen stars) can exist. Thus, the theory predicts that the whole mass spectrum of

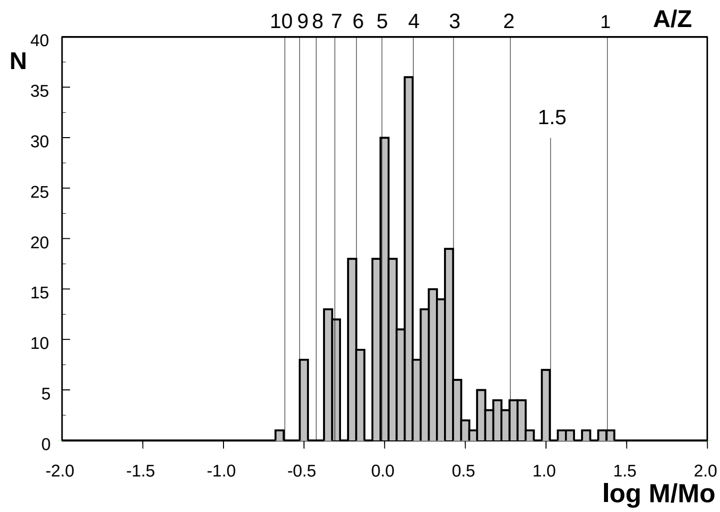

Figure 1: The mass distribution of binary stars [7]. On abscissa, the logarithm of the star mass over the Sun mass is shown. Solid lines mark masses agree with selected values of A/Z from Eq.(22)

stellar masses must be placed in the interval from few tenth up to approximately 25 solar masses. These predictions are verified by measurements quite exactly. The mass distribution of binary stars is shown on Fig.1 [7]. (Using of this data is caused by the fact that only the measurement of parameters of the binary star rotation gives the possibility to determine their masses with satisfactory accuracy). Besides, one can see the presence of separate pikes for stars with $A/Z = 3; 4; 5...$ and with $A/Z = 3/2$ on Fig.1. The stars with $A/Z = 2$ are observed too, but they don't form a separate peak on this histogram. It was mentioned above, that the chemical composition of the sunny core can be calculated [6] on base of the measurement data of natural seismic oscillations of the sunny surface [5]. This calculation shows that the core of the Sun is basically consisting from hellium-10 (A/Z=5). This is in full agreement with its position on histogram Fig.(1).

3 The mass-radius ratio

In according with the virial theorem [2, 3], the full energy of a particle inside a star must be equal to its kinetic energy with sign minus:

$$E = -\frac{3}{2}kT \tag{23}$$

In this case the heat capacity at a constant volume (per particle over Boltzman constant k) by definition is

$$c_v = -\frac{3}{2} \tag{24}$$

The negative heat capacity of stellar substance is not surprising. It is a known fact which was discussed in Landau-Lifshitz course [2]. The own heat capacity of each particle of the star substance is positive. One obtains the negative heat capacity taking into account the gravitational interaction between particles. The definition of the heat capacity of an ideal gas particle at permanent pressure [2]

$$c_p = c_v + 1 \tag{25}$$

gives

$$c_p = -\frac{1}{2} \tag{26}$$

and the adiabatic exponent

$$\widetilde{\gamma} = \frac{c_p}{c_v} = \frac{1}{3}. \tag{27}$$

Supposing that a stellar atmosphere is adiabatic, we can use the Poisson adiabat definition [2]

$$\widehat{P}V_0^{\widetilde{\gamma}} = const \tag{28}$$

and obtain

$$\widehat{P}R_0 = const. \tag{29}$$

From pressure averaged over star volume

$$\widehat{P} \approx \frac{GM^2}{R_0^4} \tag{30}$$

we obtain desired relation between mass and radius of a star:

$$\frac{M^2}{R_0^3} = const \tag{31}$$

Simultaneously the observational data of masses, of radii and their temperatures was obtained by astronomers for close binary stars. For convenience of readers, the data of these measurements for 100 stars from 50 close pairs are gathered in Table 1 in Appendix of this article. The dependence of radii of these stars over these masses is shown on Fig.2 in twice logarithmic scale. The solid line shows the result of fitting of measurement data $R \sim M^{0.68}$. It is close to theoretical dependence $R \sim M^{2/3}$ (Eq.31) which is shown by the dotted line.

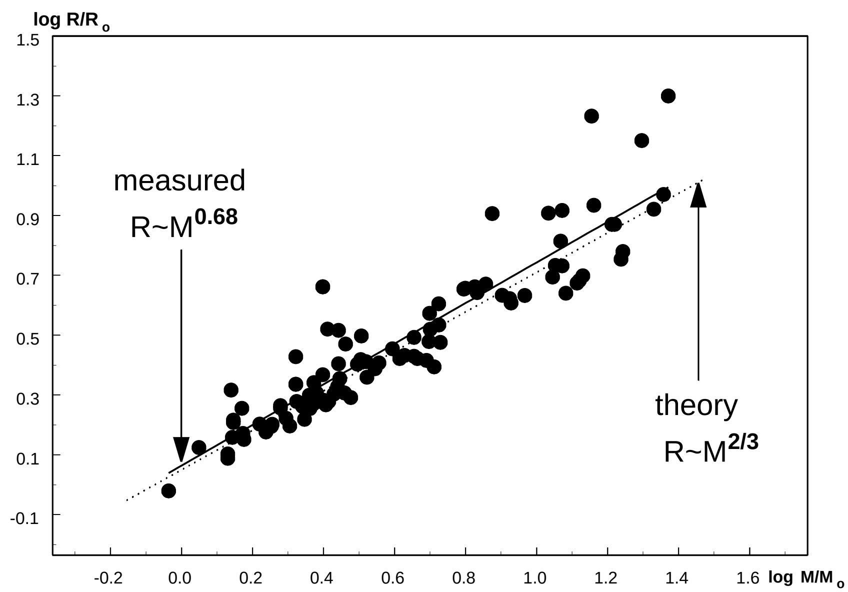

Figure 2: The dependence of radii of stars over these masses. The radii of stars are normalized to the sunny radius, the stars masses are normalized to the mass of the Sum. The data are shown in twice logarithmic scale. The solid line shows the result of fitting of measurement data ($R \sim M^{0.68}$). The theoretical dependence ($R \sim M^{2/3}$ (Eq.31)) is shown by the dotted line.

4 The mass-temperature and mass-luminosity relations

According to Eq.20 the temperature on the star surface

$$T_0 = \mathbb{T}\left(\frac{\mathbb{R}}{R_0}\right)^4. \tag{32}$$

Taking into account the characteristics of the star core (Eq.8)-(Eq.10) and (Eq.31), we obtain a relation for main parameters of star atmosphere

$$R_0 T_0 \sim \frac{1}{(1+Z)^3} \tag{33}$$

If we suppose as above that a star atmosphere is in adiabatic conditions, its averaged parameters can be related by the Poisson adiabat equation [2]:

$$\widehat{T}^{\tilde{\gamma}} \widehat{P}^{1-\tilde{\gamma}} = const \tag{34}$$

The averaged temperature of a star atmosphere can be obtained by integrating over its volume:

$$\widehat{T} \sim T_0\left(\frac{R_0}{\mathbb{R}}\right). \tag{35}$$

Averaged parameters of substance inside a star atmosphere can be described by the ideal gas law

$$\widehat{P} = k\widehat{T}\ \widehat{n}, \tag{36}$$

where averaged particle density into atmosphere

$$\widehat{n} \approx \frac{N_A}{R_0^3}. \tag{37}$$

Taking into account (Eq.21) and (Eq.9), full number particles into atmosphere

$$N_A \sim (A/Z)^{-3/2} \tag{38}$$

we obtain

$$T_0 \sim R_0^{7/8}, \tag{39}$$

or considering (Eq.31)

$$T_0 \sim \mathbb{M}^{7/12}. \tag{40}$$

The dependence of the temperature on the star surface over the star mass for the same set of data as before on Fig.(2). Here the temperatures of stars are normalized to the sunny surface temperature (5875 C), the stars masses are normalized to the mass of the Sum. The data are shown in twice logarithmic scale. The solid line shows the result of fitting of measurement data ($T_0 \sim M^{0.59}$). The theoretical dependence $T_0 \sim M^{7/12}$ (Eq.40) is shown by the dotted line.

The luminosity of a star

$$L_0 \sim R_0^2 T_0^4. \tag{41}$$

Taking into account (Eq.31) and (Eq.40) it can be expressed as

$$L_0 \sim \mathbb{M}^{11/3} \sim \mathbb{M}^{3.67} \tag{42}$$

This dependence is shown in Fig.(4)

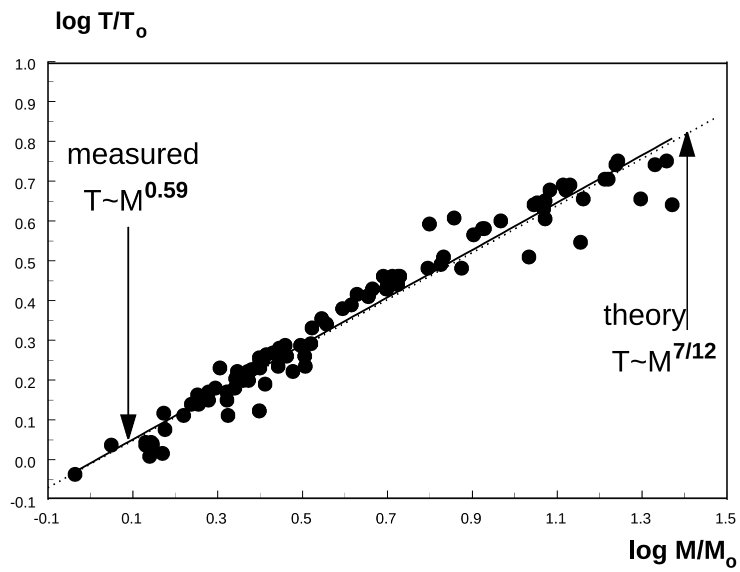

Figure 3: The dependence of the temperature on the star surface over the star mass for the same set of data as before on Fig.(2). The temperatures of stars are normalized to the sunny surface temperature (5875 C), the stars masses are normalized to the mass of the Sum. The data are shown in twice logarithmic scale. The solid line shows the result of fitting of measurement data ($T_0 \sim M^{0.59}$). The theoretical dependence $T_0 \sim M^{7/12}$ (Eq.40) is shown by the dotted line.

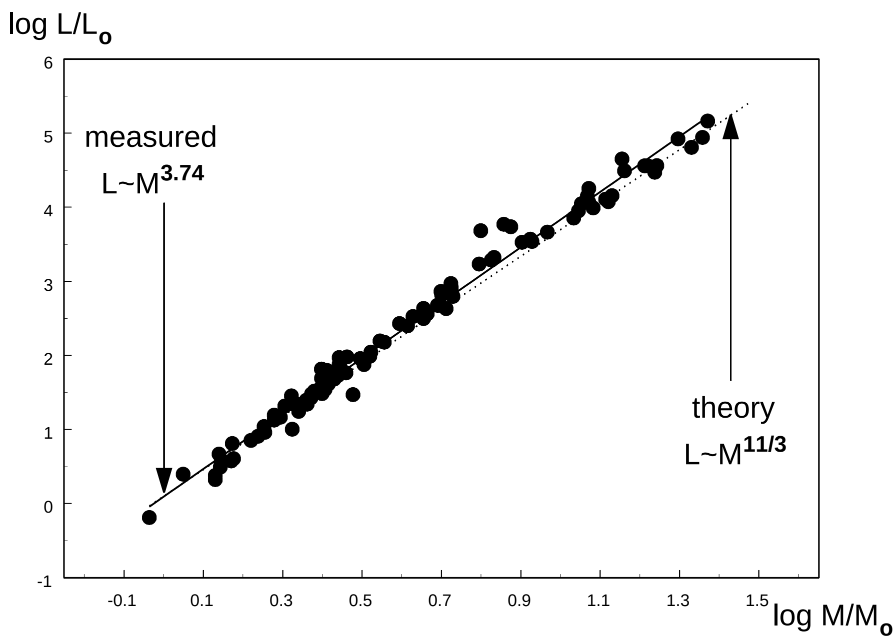

Figure 4: The dependence of star luminosity over the star mass for the same set of data as before on Fig.(2) and Fig.(3). The luminosities are normalized to the luminosity of the Sun, the stars masses are normalized to mass of the Sum. The data are shown in twice logarithmic scale. The solid line shows the result of fitting of measurement data $L \sim M^{3.74}$. The theoretical dependence $L \sim M^{11/3}$ (Eq.42) is shown by the dotted line.

5 Conclusion

It can be seen that the theoretically obtained dependencies describing the star mass distribution and mass-radius, mass-temperature and mass-luminosity relations are in a good agreement with measurement data. These theoretical results together with dependencies explaining the velocity of the periastron rotation of close binary stars [4] and explaining the spectrum of seismical oscillations of the Sun [6] are composing a practically full set of theoretical predictions that can by checked by a comparison with the measurement data. Their accordance unambiguously and confidently shows that there are cores in central regions of stars. Here the gravity force is balanced by a force with electric nature. This accordance confirms the validity of basic postulate Eq.(3).

The comparison of theoretical dependencies mass-radius, mass-temperature, mass-luminosity and measurement data became technical possible when information about main parameters of close binary stars was obtained, because it keeps the set of data about masses, radii and temperatures of stars simultaneously. This information is the result of measurements of a lot of astronomers from a lot of countries. At last time the summary table with these data was gathered by Khaliulilin Kh.F. in his dissertation (in Russian) which has unfortunately a restricted access. With his agreement and for readers convenience, we place this table in Appendix.

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6 Appendix

N	Name of star	U period of apsidal rotation, years	P period of elipsodal rotation, days	$M_1/M_\odot$ mass of component 1, in the Sun mass	$M_2/M_\odot$ mass of component 2, in the Sun mass	$R_1/R_\odot$ radius of 1 component in the Sun radius	$R_2/R_\odot$ radius of 2 component in the Sun radius	T_1 temperature of 1 component, K	T_2 temperature of 2 component, K	References
1	BW Aqr	5140	6.720	1.48	1.38	1.803	2.075	6100	6000	1,2
2	V 889 Aql	23200	11.121	2.40	2.20	2.028	1.826	9900	9400	3,4
3	V 539 Ara	150	3.169	6.24	5.31	4.512	3.425	17800	17000	5,12,24,67
4	AS Cam	2250	3.431	3.31	2.51	2.580	1.912	11500	10000	7,13
5	EM Car	42	3.414	22.80	21.40	9.350	8.348	33100	32400	8
6	GL Car	25	2.422	13.50	13.00	4.998	4.726	28800	28800	9
7	QX Car	361	4.478	9.27	8.48	4.292	4.054	23400	22400	10,11,12
8	AR Cas	922	6.066	6.70	1.90	4.591	1.808	18200	8700	14,15
9	IT Cas	404	3.897	1.40	1.40	1.616	1.644	6450	6400	84,85
10	OX Cas	40	2.489	7.20	6.30	4.690	4.543	23800	23000	16,17
11	PV Cas	91	1.750	2.79	2.79	2.264	2.264	11200	11200	18,19
12	KT Cen	260	4.130	5.30	5.00	4.028	3.745	16200	15800	20,21
13	V 346 Cen	321	6.322	11.80	8.40	8.263	4.190	23700	22400	20,22
14	CW Cep	45	2.729	11.60	11.10	5.392	4.954	26300	25700	23,24
15	EK Cep	4300	4.428	2.02	1.12	1.574	1.332	10000	6400	25,26,27,6
16	α Cr B	46000	17.360	2.58	0.92	3.314	0.955	9100	5400	28,29
17	Y Cyg	48	2.997	17.50	17.30	6.022	5.680	33100	32400	23,30
18	Y 380 Cyg	1550	12.426	14.30	8.00	17.080	4.300	20700	21600	31
19	V 453 Cyg	71	3.890	14.50	11.30	8.607	5.410	26600	26000	17,32,33
20	V 477 Cyg	351	2.347	1.79	1.35	1.567	1.269	8550	6500	34,35
21	V 478 Cyg	26	2.881	16.30	16.60	7.422	7.422	29800	29800	36,37
22	V 541 Cyg	40000	15.338	2.69	2.60	2.013	1.900	10900	10800	38,39
23	V 1143 Cyg	10300	7.641	1.39	1.35	1.440	1.226	6500	6400	40,41,42
24	V 1765 Cyg	1932	13.374	23.50	11.70	19.960	6.522	25700	25100	28
25	DI Her	29000	10.550	5.15	4.52	2.478	2.689	17000	15100	44,45,46,47
26	HS Her	92	1.637	4.25	1.49	2.709	1.485	15300	7700	48,49
27	CO Lac	44	1.542	3.13	2.75	2.533	2.128	11400	10900	50,51,52
28	GG Lup	101	1.850	4.12	2.51	2.644	1.917	14400	10500	17
29	RU Mon	348	3.585	3.60	3.33	2.554	2.291	12900	12600	54,55
30	GN Nor	500	5.703	2.50	2.50	4.591	4.591	7800	7800	56,57
31	U Oph	21	1.677	5.02	4.52	3.311	3.110	16400	15200	53,58,37
32	V 451 Oph	170	2.197	2.77	2.35	2.538	1.862	10900	9800	59,60
33	β Ori	228	5.732	19.80	7.50	14.160	8.072	26600	17800	61,62,63
34	FT Ori	481	3.150	2.50	2.30	1.890	1.799	10600	9500	64
35	AG Per	76	2.029	5.36	4.90	2.995	2.606	17000	17000	23,24
36	IQ Per	119	1.744	3.51	1.73	2.445	1.503	13300	8100	65,66
37	ζ Phe	44	1.670	3.93	2.55	2.851	1.852	14100	10500	11,67
38	KX Pup	170	2.147	2.50	1.80	2.333	1.593	10200	8100	21
39	NO Pup	37	1.257	2.88	1.50	2.028	1.419	11400	7000	11,69
40	VV Pyx	3200	4.596	2.10	2.10	2.167	2.167	8700	8700	70,71
41	YY Sgr	297	2.628	2.36	2.29	2.196	1.992	9300	9300	72
42	V 523 Sgr	203	2.324	2.10	1.90	2.682	1.839	8300	8300	73
43	V 526 Sgr	156	1.919	2.11	1.66	1.900	1.597	7600	7600	74
44	V 1647 Sgr	592	3.283	2.19	1.97	1.832	1.669	8900	8900	75
45	V 2283 Sgr	570	3.471	3.00	2.22	1.957	1.656	9800	9800	76,77
46	V 760 Sco	40	1.731	4.98	4.62	3.015	2.642	15800	15800	78
47	AO Vel	50	1.585	3.20	2.90	2.623	2.954	10700	10700	79
48	EO Vel	1600	5.330	3.21	2.77	3.145	3.284	10100	10100	21,63
49	α Vir	140	4.015	10.80	6.80	8.097	4.394	19000	19000	80,81,68
50	DR Vul	36	2.251	13.20	12.10	4.814	4.369	28000	28000	82,83

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